ДИС 26-11-12

$$\left(1+\frac{p}{n}\right)^n = e^p \qquad e^p \overset{def}{=} \sum_{k=0}^{\infty} \frac{p^k}{k!}$$

$$e = \sum_{n=1}^{\infty} \frac{1}{n!}$$

$$\left(1+\frac{p}{n}\right) = \sum_{k=0}^{n} \binom{n}{k} \frac{p^k}{n^k} = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} \frac{p^k}{n^k} < \sum_{k=0}^{n} \frac{p^k}{k!} \quad \text{при } p>0$$

$$\Rightarrow \left(1+\frac{p}{n}\right)^n < \sum_{k=0}^{n} \frac{p^k}{k!}$$

За да направим граничен преход по n в последното неравенство, трябва да знаем че границата на $\left(1+\frac{p}{n}\right)^n$ е сходяща

Док-во

от Th1 Ако a_n е ограничена и монотонно то тя е сходяща

Доказваме че е монотонна по Лагранж, т.е.

$$\frac{a_{n+1}}{a_n} \Rightarrow \frac{\left(1+\frac{p}{n+1}\right)^{n+1}}{\left(1+\frac{p}{n}\right)^n} = \frac{\left(1+\frac{p}{n+1}\right)^n \cdot \left(1+\frac{p}{n+1}\right)}{\left(1+\frac{p}{n}\right)^n} =$$

$$\left(1+\frac{p}{n+1}\right)\left[\frac{(n+1+p)n}{(n+1)(p+n)}\right]^n \Rightarrow \text{от Бернули}$$

$$(1+x)^n \geq 1+nx \Rightarrow$$

$$\frac{1+\frac{p}{n+1}}{1-\frac{1}{(n+1)(p+n)}} = \frac{\frac{(n+p)(n+p-1)}{n(n+p)}}{\frac{(n+1)(n+p)-1}{(n+1)(n+p)}} \Rightarrow$$

$\exists \lim \left(1+\frac{p}{n}\right)^n$ т.е. $\sum_{k=0}^{m} \frac{p^k}{k!} \le \lim \left(1+\frac{p}{n}\right)^n$

искомое $m < n$, следовательно

$$\sum_{k=0}^{m} \left(1-\frac{1}{n}\right)\cdots\left(1-\frac{k-1}{n}\right)\frac{p^k}{k!} \le e^p \Rightarrow$$

$$e^p \le \lim \left(1+\frac{p}{n}\right)^n \le e^p \quad \text{(двойная помощь)}$$

Счетное $\lim$

$$\lim \left(\frac{n}{n+1}\right)^{3n} = \lim \left(\frac{n+1-1}{n+1}\right)^{3n} = \lim \left(1-\frac{1}{n+1}\right)^{3n} =$$

$$\lim \left[\left(1-\frac{1}{n+1}\right)^{n+1-1}\right]^3 = \lim \underbrace{\left[\left(1+\frac{-1}{n+1}\right)^{n+1}\right]^3}_{e^{-1}} \left(1-\frac{1}{n+1}\right)^{-3} =$$

$$(e^{-1})^3 = e^{-3}$$

$$\lim \left(\frac{3n-2}{3n+1}\right)^{2n} = \lim \sqrt[3]{\left(1+\frac{-3}{3n+1}\right)^{3n}}^{2} \Rightarrow$$

от $a^{\frac{m}{n}} \overset{def}{=} \sqrt[n]{a^m}$, $a>0$ $\Rightarrow$ след ± 1 в степен

$$\lim \sqrt[3]{\frac{\left(1+\frac{-3}{3n+1}\right)^{n+1}}{\left(1+\frac{-3}{3n+1}\right)}}^{2} = \lim \frac{\sqrt[3]{\left(1+\frac{-3}{3n+1}\right)^{3n+1}}^{2}}{\sqrt[3]{1-\frac{3}{3n+1}}^{2}} =$$

$$\sqrt[3]{e^{-3}}^{2} = \frac{1}{e^2} = e^{-2}$$

$$\lim \left(1+\frac{1}{n^3}\right)^{n^2+1} = \lim \sqrt[n]{\left(1+\frac{1}{n^3}\right)^{n^3}} \left(1+\underbrace{\frac{1}{n^3}}_{0}\right) =$$

Ако $a_n \to a > 0$ то $\sqrt[n]{a_n} \to 1$

$\lim a_n = \lim \left(1+\frac{1}{n^3}\right)^{n^3} = e > 0 \Rightarrow \to 1$

$$\lim \left(1+\frac{1}{n^3}\right)^{n^2+1} = 1$$

Ако $\alpha_n \to +\infty$, $\alpha_n \in \mathbb{Q}$ то $\left(1+\frac{p}{\alpha_n}\right)^{\alpha_n} \to e^p$

За $\forall n \in \mathbb{N}$ $\exists k_n \leq \alpha_n \leq k_{n+1}$ $(k_n \in \mathbb{N} \to \infty)$

$$\left(1+\frac{1}{k_n+1}\right)^{k_n} < \left(1+\frac{1}{k_n+1}\right)^{\alpha_n} \le \left(1+\frac{1}{\alpha_n}\right)^{\alpha_n} \le \left(1+\frac{1}{k_n}\right)^{\alpha_n}$$

забимаме | ако $m<n$ то $a^m < a^n$ за $a>1$
ако $a<b$ то $a^z < b^z$ за $z \in Q$ | $\Rightarrow$

$$< \left(1+\frac{1}{k_n}\right)^{k_n+1}$$

$$\frac{\left(1+\frac{1}{k_n+1}\right)^{k_n+1}}{\left(1+\frac{1}{k_n}\right)} < \left(1+\frac{1}{\alpha_n}\right)^{\alpha_n} < \left(1+\frac{1}{k_n}\right)^{k_n} < \left(1+\frac{1}{k_n}\right)$$

$$e \le \lim \left(1+\frac{1}{\alpha_n}\right)^{\alpha_n} \le e$$